


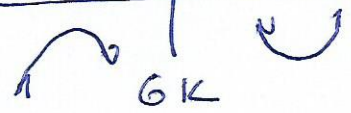
ΘΕΜΑ Β

B₁) Θετω $x+1=u \Leftrightarrow x=u-1$
 $f(u) = (u-1+1)e^{-(u-1)} = u \cdot e^{1-u}, u \in \mathbb{R}$
 $f(x) = x \cdot e^{1-x}, x \in \mathbb{R}$

B₂) $f'(x) = e^{1-x} - x e^{1-x} = e^{1-x} (1-x)$

x	$-\infty$	1	$+\infty$
$f'(x)$		+	-
$f(x)$			

B₃) $f''(x) = -e^{1-x} (1-x) - e^{1-x} = -e^{1-x} (2-x)$

x	$-\infty$	2	$+\infty$
$f''(x)$		-	+
$f(x)$			

$f(2) = 2 \cdot e^{-1} = \frac{2}{e}$

$\lim_{x \rightarrow +\infty} f(x) = \lim_{x \rightarrow +\infty} \frac{x}{e^{x-1}} \stackrel{+\infty}{=} \lim_{x \rightarrow +\infty} \frac{1}{e^{x-1}} = 0$
D.L.N

$y=0$ οριζόντια ασύμπτωτη
 στο $+\infty$

$$\lim_{x \rightarrow -\infty} f(x) = -\infty, \quad \lim_{x \rightarrow -\infty} \frac{f(x)}{x} = \lim_{x \rightarrow -\infty} e^{1-x} = +\infty$$

Άρα δεν υπάρχουν ασύμπτωτες
στο $-\infty$.

$$B_4) \text{ i) } f(-\infty, 1) \xrightarrow{\uparrow} R_1 = \left(\lim_{x \rightarrow -\infty} f(x), \lim_{x \rightarrow 1^-} f(x) \right) =$$

$$= (-\infty, 1)$$

$$f: [1, +\infty) \xrightarrow{\downarrow} \left(\lim_{x \rightarrow +\infty} f(x), f(1) \right) = (0, 1]$$

$$R_f = R_1 \cup R_2 = (-\infty, 1]$$

$$ii) f(x) = \lambda$$

$$A_{\lambda} \quad \lambda \leq 0 \quad 1 \text{ ρίζα}$$

$$A_{\lambda} \quad \lambda \in (0, 1) \quad 2 \text{ ρίζες}$$

$$A_{\lambda} \quad \lambda = 1 \quad 1 \text{ ρίζα}$$

$$A_{\lambda} \quad \lambda > 1 \quad 0 \text{ ρίζες}$$