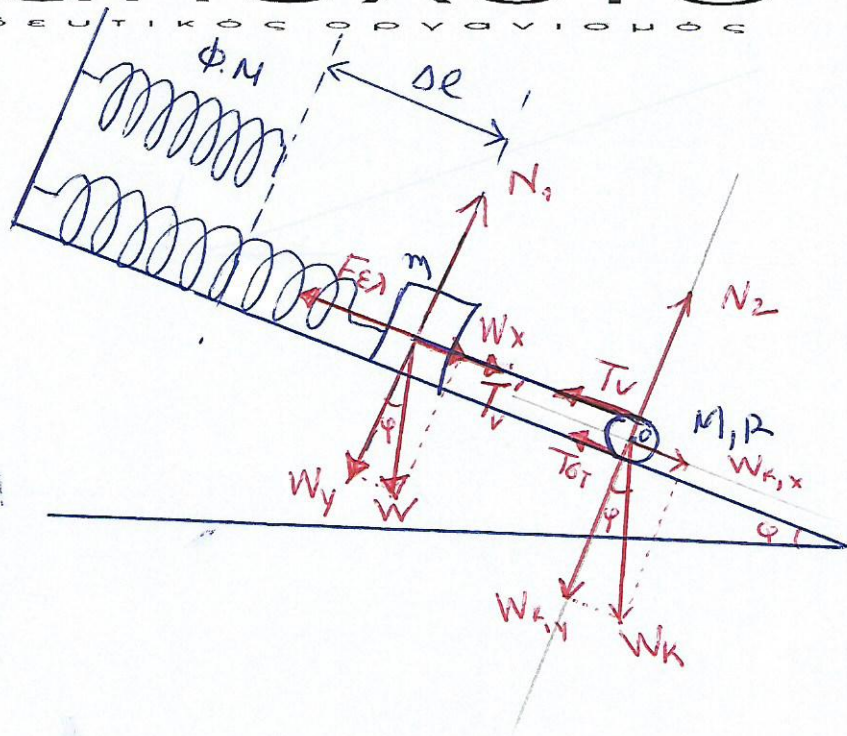


ΘΕΜΑ Δ

| Ζ                      | Δ                                  |
|------------------------|------------------------------------|
| Δ1) $T_V$              | $m = 1 \text{ kg}$                 |
| $\Delta \ell$          | $k = 100 \text{ N/m}$              |
|                        | $\varphi = 30^\circ$               |
| Δ2) $F_{\text{ελ}}(t)$ | $M = 2 \text{ kg}$                 |
| Δ3) $L$                | $R = 0,1 \text{ m}$                |
|                        | $N = \frac{12}{7}$                 |
| Δ4) $\frac{dk}{dt}$    | $t = 3 \text{ s}$                  |
|                        | $I_{\text{cm}} = \frac{1}{2} MR^2$ |



Δ1) Το σύστημα ισορροπεί άρα:

για τον κύλινδρο :

$$\bullet \sum F_x = 0 \Rightarrow W_{k,x} - T_V - T_{\sigma\tau} = 0 \Rightarrow T_{\sigma\tau} = Mg \eta \mu 30 - T_V$$

και

$$\bullet \sum \tau_0 = 0 \Rightarrow T_V \cdot R - T_{\sigma\tau} \cdot R = 0 \Rightarrow T_{\sigma\tau} = T_V$$

$$Mg \eta \mu 30 - T_V = T_V \Rightarrow T_V = \frac{Mg \eta \mu 30}{2} \Rightarrow \boxed{T_V = 5 \text{ N}}$$

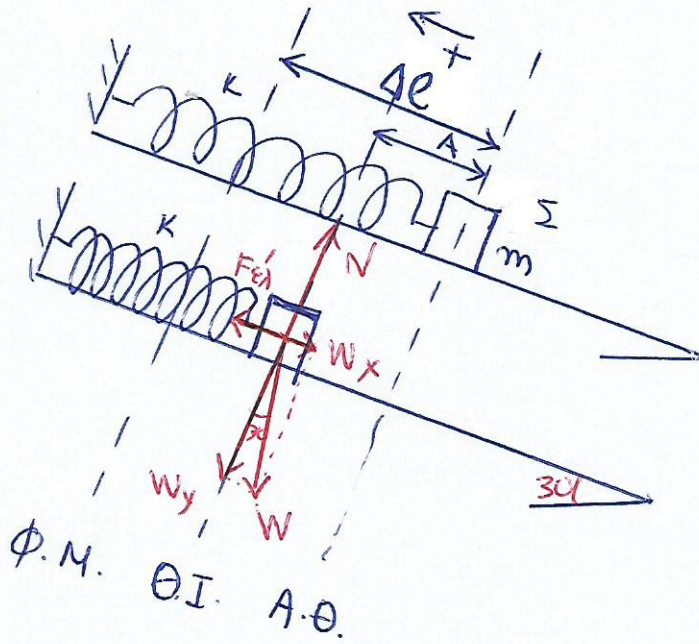
για το Σ :

$$\bullet \sum F_x = 0 \Rightarrow W_x + T_V' - F_{\text{ελ}} = 0 \xrightarrow{T_V' = T_V} F_{\text{ελ}} = mg \eta \mu 30 + T_V$$

$$\Rightarrow k \cdot \Delta \ell = mg \eta \mu 30 + T_V \Rightarrow \Delta \ell = \frac{mg \eta \mu 30 + T_V}{k} \Rightarrow \boxed{\Delta \ell = 0,1 \text{ m}}$$



Δ2



$$t=0, \text{ Α.Θ. } (x=-A)$$

Θ.Ι

• ΣΤΗΝ Θ.Ι:  $\sum F_x = 0 \Rightarrow F_{ελ}' = W_x \Rightarrow k \cdot (\Delta \ell - A) = m \cdot g \cdot \eta \mu 30$   
 $\Rightarrow k \Delta \ell - kA = mg \eta \mu 30 \Rightarrow A = \frac{k \Delta \ell - mg \eta \mu 30}{k}$   
 $\Rightarrow A = \frac{100 \text{ N/m} \cdot 0,1 \text{ m} - 1 \text{ kg} \cdot 10 \frac{\text{m}}{\text{s}^2} \cdot \frac{1}{2}}{100 \text{ N/m}} \Rightarrow \boxed{A = 0,05 \text{ m}}$

•  $x = A \eta \mu(\omega t + \varphi_0) \xrightarrow[t = -A]{t=0} \eta \mu \varphi_0 = -1 = \eta \mu \frac{3\pi}{2} \Rightarrow$   
 $\left. \begin{array}{l} \varphi_0 = 2k\pi + \frac{3\pi}{2} \\ \varphi_0 = 2k\pi + \pi - \frac{3\pi}{2} \end{array} \right\} \xrightarrow{k=0} \begin{array}{l} \boxed{\varphi_0 = \frac{3\pi}{2}} \\ \varphi_0 = -\frac{\pi}{2} \end{array}$

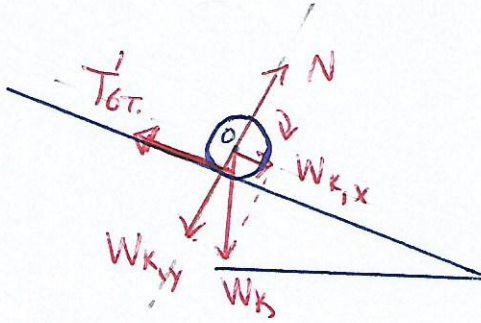
•  $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{100 \text{ N/m}}{1 \text{ kg}}} \Rightarrow \boxed{\omega = 10 \text{ rad/s}}$



$$\Sigma F = -k \cdot x$$

$$\Sigma F = -k \cdot A \eta\mu(\omega t + \varphi_0) \Rightarrow \boxed{\Sigma F = -5 \eta\mu\left(10t + \frac{3\pi}{2}\right)}$$

Δ3)



Ο κύλινδρος κάνει μεταφορική και περιστροφική κίνηση:

$$\bullet \Sigma \tau_0 = I_{cm} \alpha_{\gamma\omega\nu} \Rightarrow T'_{στ} \cdot R = \frac{1}{2} M R^2 \alpha_{\gamma\omega\nu} \Rightarrow \alpha_{\gamma\omega\nu} = \frac{\alpha_{cm}}{R}$$

κλίση χωρίς ολίσθηση άρα  $\alpha_{\gamma\omega\nu} = \frac{\alpha_{cm}}{R}$

$$T'_{στ} = \frac{1}{2} M R \cdot \frac{\alpha_{cm}}{R} \Rightarrow T'_{στ} = \frac{M \cdot \alpha_{cm}}{2} \quad \left. \vphantom{T'_{στ}} \right\} \Rightarrow$$

$$\bullet \Sigma F_x = M \cdot \alpha_{cm} \Rightarrow W_{k,x} - T'_{στ} = M \cdot \alpha_{cm}$$

$$M \cdot g \cdot \eta\mu 30 - \frac{M \cdot \alpha_{cm}}{2} = M \cdot \alpha_{cm} \Rightarrow \alpha_{cm} = \frac{2g \eta\mu 30}{3} \Rightarrow$$

$$\boxed{\alpha_{cm} = \frac{10}{3} \text{ m/s}^2} \quad \text{και} \quad \alpha_{\gamma\omega\nu} = \frac{\alpha_{cm}}{R} \Rightarrow \boxed{\alpha_{\gamma\omega\nu} = \frac{100}{3} \text{ rad/s}^2}$$

Όταν έχει περιτράψει κατά  $N$  φορές, θα έχει διαγράψει

$$N = \frac{\theta}{2\pi} \Rightarrow \theta = N \cdot 2\pi = \frac{12}{\pi} \cdot 2\pi \Rightarrow$$

$$\boxed{\theta = 24 \text{ rad}}$$

$$\theta = \frac{1}{2} \alpha_{\gamma\omega\nu} \cdot t^2 \Rightarrow t = \sqrt{\frac{2\theta}{\alpha_{\gamma\omega\nu}}} \Rightarrow \boxed{t = 1,2 \text{ sec}}$$



Άρα  $L = I_{cm} \cdot \omega$

$$L = \frac{1}{2} M \cdot R^2 \cdot \alpha_{\gamma\omega\nu} \cdot t = \frac{1}{2} 2\text{kg} (0,1\text{m})^2 \cdot \frac{100\text{rad}}{3} \cdot \frac{1,25}{\text{s}^2}$$

$$\Rightarrow \boxed{L = 0,4 \text{ kg} \frac{\text{m}^2}{\text{s}}}$$

Δ4)

$$\frac{dK}{dt} = \sum F \cdot v_{cm} + \sum \tau \cdot \omega$$

$$\frac{dK}{dt} = M \cdot a_{cm} \cdot a_{cm} \cdot t + I_{cm} \cdot \alpha_{\gamma\omega\nu} \cdot \alpha_{\gamma\omega\nu} \cdot t$$

$$\frac{dK}{dt} = M a_{cm}^2 \cdot t + \frac{1}{2} M R^2 \cdot \frac{a_{cm}^2}{R^2} \cdot t$$

$$\frac{dK}{dt} = \frac{3 M a_{cm}^2 t}{2}$$

$$\frac{dK}{dt} = \frac{3 \cdot 2\text{kg} \cdot \left(\frac{10}{3} \frac{\text{m}}{\text{s}}\right)^2 \cdot 3\text{s}}{2}$$

$$\boxed{\frac{dK}{dt} = 100 \text{ J/s}}$$