ΘΕΜΑ Δ $\qquad f(x) = (x-1)\cdot \ln(x^2-2x+2)+\alpha x+\beta$

Δ1) $f(1) = 1 \Rightarrow \alpha + \beta = 1$

$$f'(x) = \ln(x^2-2x+2) + (x-1)\frac{2(x-1)}{x^2-2x+2} + \alpha$$

$$f'(1) = -1 \Rightarrow \boxed{\alpha = -1} \qquad \boxed{\beta = 2}$$

Δ2) $f(x) = (x-1)\ln(x^2-2x+2) - x + 2$

$$\underbrace{f(x) - (-x+2)}_{h(x)} = (x-1)\cdot \ln(x^2-2x+2)$$

x		1	
$x-1$	$-$	0	$+$
$\ln(x^2-2x+2)$	$+$		$+$
$h(x)$	$-$		$+$

$\circledast$ $\ln(x^2-2x+2) \geq 0 \Leftrightarrow x^2-2x+2 \geq 1$ ισχύει πάντα

$$E(\Omega)=\int_1^2 |h(x)|\,dx=\int_1^2 (x-1)\ln(x^2-2x+2)\,dx$$

$$=\int_1^2 \frac{1}{2}(2x-2)\ln(x^2-2x+2)\,dx$$

$x^2-2x+2=u$ τότε $(2x-2)\,dx=du$

x	1	2
u	1	2

$$=\frac{1}{2}\int_1^2 \ln u\,du=\frac{1}{2}\Big[u\ln u-u\Big]_1^2=$$

$$=\frac{1}{2}\Big[(2\ln 2-2)+1\Big]=\frac{1}{2}(2\ln 2-1)=\ln 2-\frac{1}{2}$$

τ.μ.

Δ3) $f'(x) = \ln(x^2-2x+2) + \dfrac{2(x-1)^2}{x^2-2x+2} - 1$

$= \ln(x^2-2x+2) + \dfrac{2x^2-4x+2-x^2+2x-2}{x^2-2x+2}$

$= \ln(x^2-2x+2) + \dfrac{x^2-2x}{x^2-2x+2}$.

i) ΑΝΔ $f'(x) \geq -1 \Leftrightarrow \ln(x^2-2x+2) + \dfrac{2(x-1)^2}{x^2-2x+2} \geq 0$

ισχύει

ii) $f(\lambda+\frac{1}{2}) - \underbrace{(\lambda-1)\ln(\lambda^2-2\lambda+2)}_{f(\lambda)+\lambda-2} + \lambda \geq \dfrac{3}{2}$

$f(\lambda+\frac{1}{2}) - f(\lambda) \geq \dfrac{3}{2} - 2$

$f(\lambda+\frac{1}{2}) - f(\lambda) \geq -\dfrac{1}{2}$

$\dfrac{f(\lambda+\frac{1}{2}) - f(\lambda)}{\frac{1}{2}} \geq -1$

Για $\lambda \in \mathbb{R}$ $\quad$ f συνεχής $\left[\lambda, \lambda+\frac{1}{2}\right]$ $\quad$ $\overset{\text{ΘΜΤ}}{\Longrightarrow}$

f παραγ. $\left(\lambda, \lambda+\frac{1}{2}\right)$

Θα υπάρχει 1 τουλ. $\xi \in \left(\lambda, \lambda+\frac{1}{2}\right)$:

$$f'(\xi) = \frac{f\left(\lambda+\frac{1}{2}\right) - f(\lambda)}{\frac{1}{2}} \geq -1.$$

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Δ4) $f'(x) \geq -1$

$g'(x) = -3x^2 - 1 \leq -1$

εφαπτ. της C_f: $y - f(x_1) = f'(x_1)(x - x_1)$

εφαπτ. της C_g: $y - g(x_2) = g'(x_2)(x - x_2)$

$f'(x_1) = g'(x_2) = -1 \Rightarrow \begin{array}{l} x_2 = 0 \\ x_1 = 1 \end{array}$

$y - 2 = -1 \cdot x \Leftrightarrow y = -x + 2$